

FIRST NAME: \_\_\_\_\_

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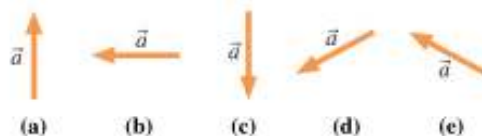
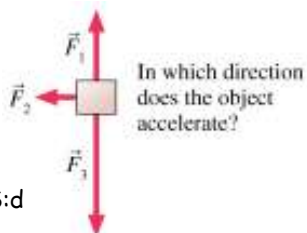
STUDENT NUMBER: \_\_\_\_\_

Closed-book Test. Duration: 110 minutes

**PART 1 MULTIPLE CHOICE QUESTIONS:** 48% of your test grade

ANSWER THESE QUESTIONS USING SCANTRON SHEET

All questions are of the same point value. Attempt all questions. **Best 6 count toward the grade**



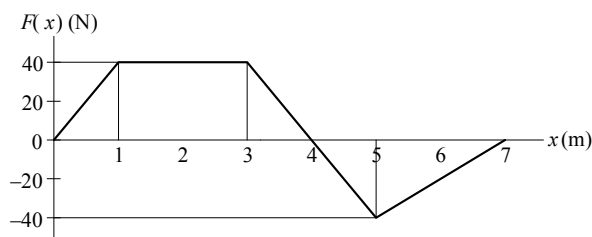
ANS: d

2. A body moves in a circle at constant speed. The work done on the body by the centripetal force in one revolution is

- a. 0.      b.  $2\pi mv^2$       c.  $Ft$       d.  $\frac{v^2}{r}$       e.  $\frac{mv^2}{r}$ .

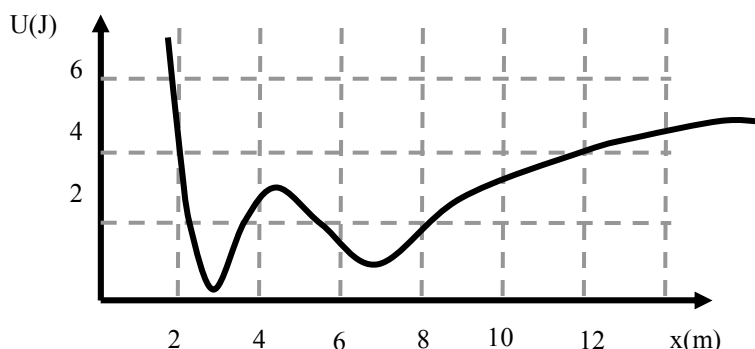
3. An object of mass 2 kg moving at  $v_x = +1\text{ m/s}$  is subjected to a single force  $F(x)$  shown in the diagram below. What is its kinetic energy when it is at position  $x=4\text{ m}$ ?

- a. -120  
b. 0  
c. 1  
d. 120  
e. 121



4. Particle of mass 1 kg moves under influence of conservative force, whose potential energy is shown in the diagram. At  $t=0$  particle at  $x=8\text{ m}$  is moving to the left and has 2 J of kinetic energy. Where is the right turning point of the particle?

- A 12m      B 2m      C 4 m  
D infinity      E none of the above



5. A 4.0 kg particle is moving horizontally with a speed of 5.0 m/s when it strikes a vertical wall. The particle rebounds with a speed of 3.0 m/s. What is the magnitude of the impulse in  $\text{N} \cdot \text{s}$  delivered to the particle?

- a. 24      b. 32      c. 40      d. 30      e. 8.0

6. A 12 g bullet is fired into a 3.0 kg ballistic pendulum initially at rest and becomes embedded in it. The pendulum subsequently rises a vertical distance of 12 cm. What was the initial speed of the bullet in km/s?

- a. 0.024      b. 0.38      c. 0.44      d. 0.50      e. 0.54

7. An explosion in a rigid pipe shoots out three pieces. A 10g piece comes out the right end. A 4 g piece comes out the left end with twice the speed of the 10 g piece. From which end does the third piece emerge?

- A. Right end  
B. Left end  
C. The third piece will not emerge from the pipe  
D. Depending on the geometry of this situation A B C are all possible  
E. Not enough information to answer this question

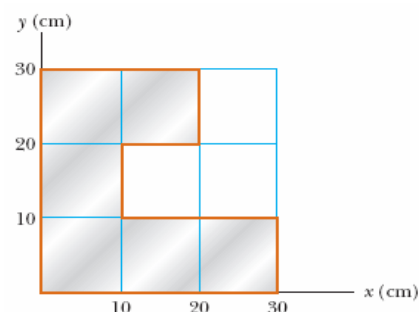
PART II (52% of the midterm test total score)

Provide full answers to four out of five problems below.

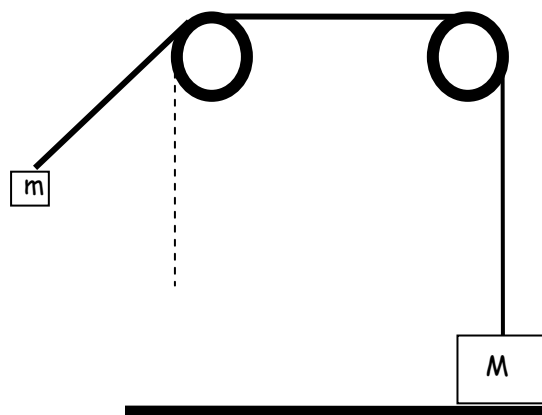
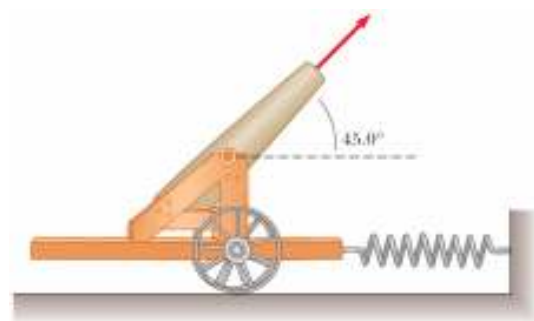
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- presentation is important ( up to 5% deduction)



- 1 A uniform piece of sheet steel is shaped as shown. Find the  $x$  and  $y$  coordinates of the center of mass of the piece.
- 2 A skier of mass 70.0 kg is pulled up a slope by a motor-driven cable.  
 (a) How much work is required to pull the skier a distance of 60.0 m up a  $30.0^\circ$  slope (assumed frictionless) at a constant speed of 2.00 m/s?  
 (b) A motor of what power is required to perform this task?
- 3 A cannon is rigidly attached to a carriage, which can move along horizontal rails but is connected to a post by a large spring, initially un-stretched and with force constant  $k = 2.00 \times 10^4$  N/m, as shown. The cannon fires a 200-kg projectile at a velocity of 125 m/s directed  $45.0^\circ$  above the horizontal.  
 (a) Assuming that the mass of the cannon and its carriage is 5 000 kg, find the recoil speed of the cannon. (4p)  
 (b) Determine the maximum extension of the spring. (3p)  
 (c) Find the maximum force the spring exerts on the carriage. (3p)  
 (d) Consider the system consisting of the cannon, carriage, and projectile. Is the linear momentum of this system conserved during the firing? Why or why not? (3p)
- 4 A bullet of mass  $m=10\text{g}$  and speed  $=1000\text{m/s}$  hits block of  $M=10\text{kg}$  resting on the frictionless surface. After the collision the block and the bullet embedded in it enter the rough patch of the surface. And travel distance  $x$  before coming to rest.  
 a) Find the  $x$  (8p)  
 b) What percentage of the energy was lost in this collision? (5p)
- 5 The mass  $m=1$  kg is set up as a pendulum of length  $L=0.6\text{m}$  as shown on the diagram. Initially in the position of  $h$  above its lowest point, the pendulum is at rest. Knowing that the largest mass  $M$  that could be temporarily lifted in such setup is  $M=2.4\text{kg}$  find  $h$ .



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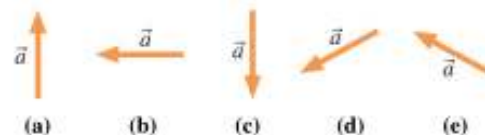
1 Three forces act on an object.

In which direction does the object accelerate?

ANS e)



In which direction does the object accelerate?



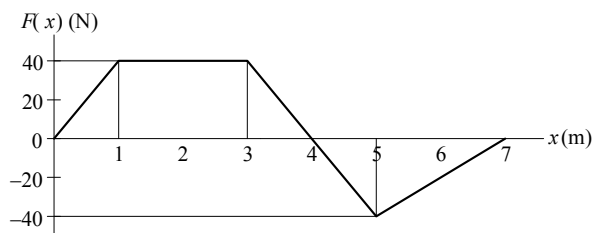
2. A body moves in a circle at constant speed. The work done on the body by the centripetal force in one revolution is

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d.  $\frac{mv^2}{r}$       e. 0

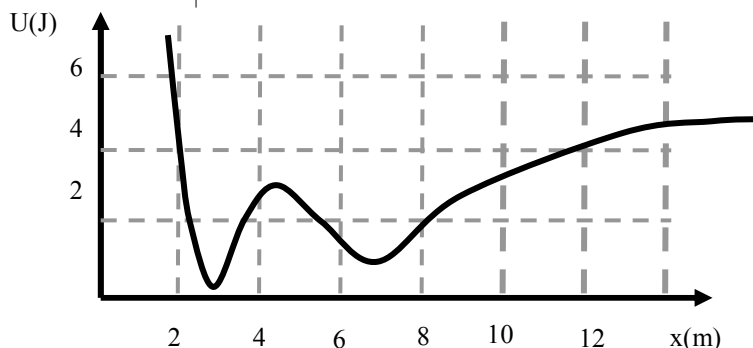
3. An object of mass 2 kg moving at  $v_x = +1\text{m/s}$  is subjected to a single force  $F(x)$  shown in the diagram below. What is its kinetic energy when it is at position  $x=7\text{m}$ ?

a. -20      b. 1      c. 61  
d. 21      e. 121



4 Particle of mass 1kg moves under influence of conservative force, whose potential energy is shown in the diagram. At  $t=0$  particle at  $x=8\text{m}$  is moving to the left and has 2 J of kinetic energy. Where is the left turning point of the particle?

A 12m      B 4m      C 2 m  
D infinity      E none of the above



5. A 2.0 kg particle is moving horizontally with a speed of 8.0 m/s when it strikes a vertical wall. The particle rebounds with a speed of 4.0 m/s. What is the magnitude of the impulse in  $\text{N} \cdot \text{s}$  delivered to the particle?

a. 24      b. 32      c. 40      d. 30      e. 8.0

6 A 12 g bullet is fired into a 3.0 kg ballistic pendulum initially at rest and becomes embedded in it. The pendulum subsequently rises a vertical distance of 12 cm. What was the initial speed of the bullet in km/s?

a. 0.24      b. 0.32      c. 0.38      d. 0.50      e. 0.54

7 An explosion in a rigid pipe shoots out three pieces. A 6 g piece comes out the right end. A 4 g piece comes out the left end with three halves of the speed of the 6 g piece. From which end does the third piece emerge?

A Right end  
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C The third piece will not emerge from the pipe  
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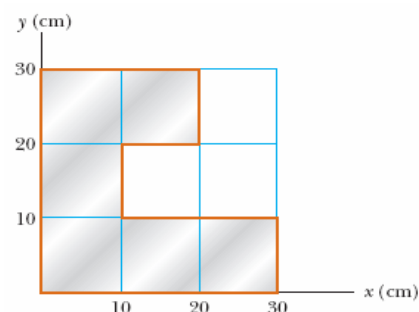
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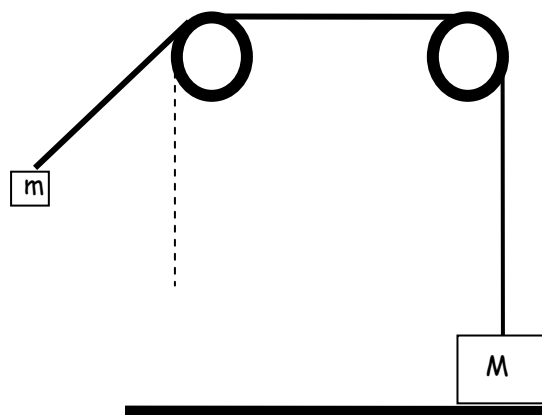
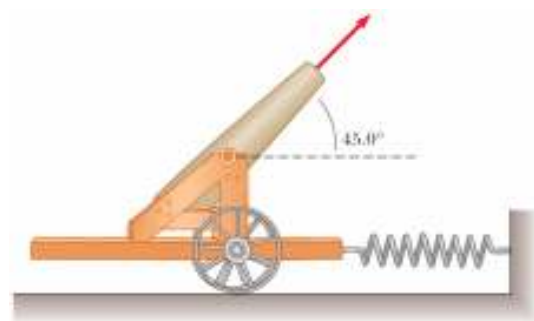
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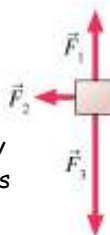
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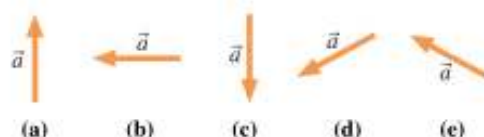
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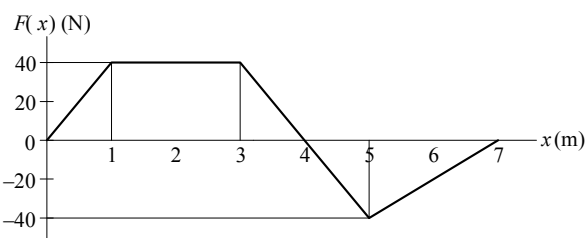


In which direction does the object accelerate?



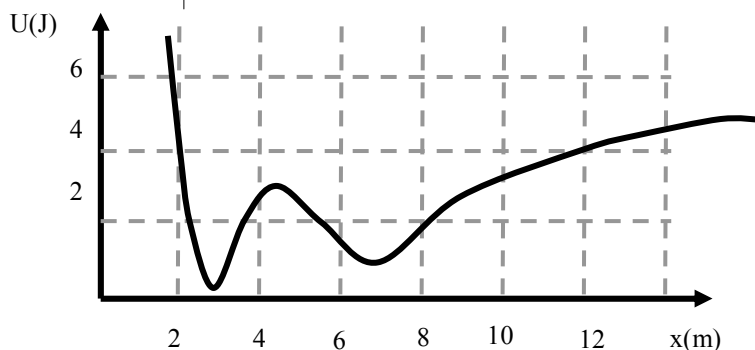
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- A. 12 m      B. 2 m      C. 4 m  
D. infinity      E. none of the above



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6. A 12 g bullet is fired into a 3.0 kg ballistic pendulum initially at rest and becomes embedded in it. The pendulum subsequently rises a vertical distance of 12 cm. What was the initial speed of the bullet in  $\text{km/s}$ ?

- a. 0.380      b. 0.42      c. 0.44      d. 0.50      e. 0.54

7. An explosion in a rigid pipe shoots out three pieces. A 10 g piece comes out the right end. A 4 g piece comes out the left end with twice the speed of the 10 g piece. From which end does the third piece emerge?

- F. Left end  
G. Right end  
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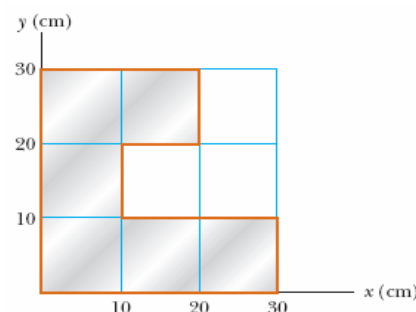
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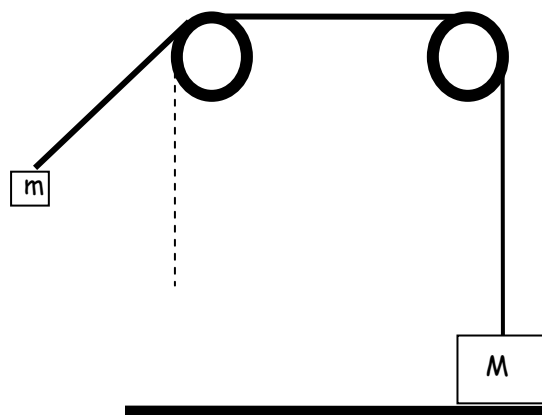
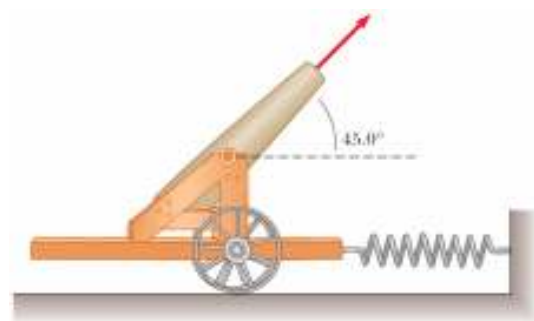
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**ANSWERS:****These answers only work with one set of numbers****PROBLEM 1** Let  $A_1$  represent the area of the bottom row of squares,  $A_2$  the middle square, and  $A_3$  the top pair.

$$A = A_1 + A_2 + A_3$$

$$M = M_1 + M_2 + M_3$$

$$\frac{M_1}{A_1} = \frac{M}{A}$$

$$A_1 = 300 \text{ cm}^2, A_2 = 100 \text{ cm}^2, A_3 = 200 \text{ cm}^2, A = 600 \text{ cm}^2$$

$$M_1 = M \left( \frac{A_1}{A} \right) = \frac{300 \text{ cm}^2}{600 \text{ cm}^2} M = \frac{M}{2}$$

$$M_2 = M \left( \frac{A_2}{A} \right) = \frac{100 \text{ cm}^2}{600 \text{ cm}^2} M = \frac{M}{6}$$

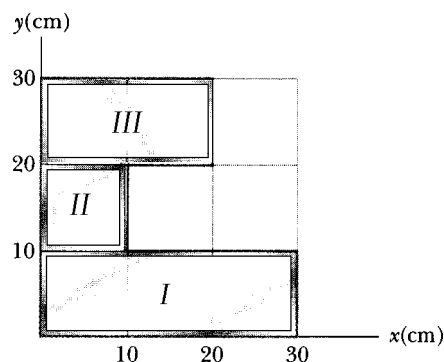
$$M_3 = M \left( \frac{A_3}{A} \right) = \frac{200 \text{ cm}^2}{600 \text{ cm}^2} M = \frac{M}{3}$$

$$x_{\text{CM}} = \frac{x_1 M_1 + x_2 M_2 + x_3 M_3}{M} = \frac{15.0 \text{ cm} \left( \frac{1}{2} M \right) + 5.00 \text{ cm} \left( \frac{1}{6} M \right) + 10.0 \text{ cm} \left( \frac{1}{3} M \right)}{M}$$

$$x_{\text{CM}} = \boxed{11.7 \text{ cm}}$$

$$y_{\text{CM}} = \frac{\frac{1}{2} M (5.00 \text{ cm}) + \frac{1}{6} M (15.0 \text{ cm}) + \left( \frac{1}{3} M \right) (25.0 \text{ cm})}{M} = 13.3 \text{ cm}$$

$$y_{\text{CM}} = \boxed{13.3 \text{ cm}}$$



## PROBLEM 2

- (a)  $\sum W = \Delta K$ , but  $\Delta K = 0$  because he moves at constant speed. The skier rises a vertical distance of  $60.0 \text{ m} \sin 30.0^\circ = 30.0 \text{ m}$ . Thus,

$$W_{\text{in}} = -W_g = (70.0 \text{ kg})(9.8 \text{ m/s}^2)(30.0 \text{ m}) = \boxed{2.06 \times 10^4 \text{ J}} = \boxed{20.6 \text{ kJ}}.$$

- (b) The time to travel 60.0 m at a constant speed of 2.00 m/s is 30.0 s. Thus,

$$P_{\text{input}} = \frac{W}{\Delta t} = \frac{2.06 \times 10^4 \text{ J}}{30.0 \text{ s}} = \boxed{686 \text{ W}} = 0.919 \text{ hp}.$$



### PROBLEM 3

- (a) Use conservation of the horizontal component of momentum for the system of the shell, the cannon, and the carriage, from just before to just after the cannon firing.

$$p_{xf} = p_{xi}: \quad m_{\text{shell}} v_{\text{shell}} \cos 45.0^\circ + m_{\text{cannon}} v_{\text{recoil}} = 0$$

$$(200)(125) \cos 45.0^\circ + (5\,000) v_{\text{recoil}} = 0$$

or  $v_{\text{recoil}} = \boxed{-3.54 \text{ m/s}}$

- (b) Use conservation of energy for the system of the cannon, the carriage, and the spring from right after the cannon is fired to the instant when the cannon comes to rest.

$$K_f + U_{gf} + U_{sf} = K_i + U_{gi} + U_{si}: \quad 0 + 0 + \frac{1}{2} k x_{\text{max}}^2 = \frac{1}{2} m v_{\text{recoil}}^2 + 0 + 0$$

$$x_{\text{max}} = \sqrt{\frac{m v_{\text{recoil}}^2}{k}} = \sqrt{\frac{(5\,000)(-3.54)^2}{2.00 \times 10^4}} \text{ m} = \boxed{1.77 \text{ m}}$$

(c)  $|F_{s, \text{max}}| = k x_{\text{max}} \quad |F_{s, \text{max}}| = (2.00 \times 10^4 \text{ N/m})(1.77 \text{ m}) = \boxed{3.54 \times 10^4 \text{ N}}$

- (d) No. The rail exerts a vertical external force (the normal force) on the cannon and prevents it from recoiling vertically.  
Momentum is not conserved in the vertical direction. The spring does not have time to stretch during the cannon firing.  
Thus, no external horizontal force is exerted on the system (cannon, carriage, and shell) from just before to just after firing.  
Momentum of this system is conserved in the horizontal direction during this interval.

Answer 4

This is one of the scenarios for the ballistic pendulum

a) the collision is inelastic -- we may use the conservation of linear momentum to establish the speed of the block after the collision

$$mv = (M + m)v' \Rightarrow v' = \frac{m}{(M + m)}v = \frac{0.01}{10.01}1000 \frac{m}{s} = 0.999 \frac{m}{s}$$

The kinetic energy of the block with bullet in it will be completely lost on the rough patch due to the force of friction.

Since  $|K_i| = |E_{\text{loss}}| = |W|$

$$\frac{(m + M)v'^2}{2} = W = fx = \mu(M + m)gx \Rightarrow x = \frac{v'^2}{2\mu g(M + m)} = \frac{0.998}{2(0.03)(9.8)(10.01)} = 0.17m$$

b)

$$\frac{\frac{(m + M)v'^2}{2}}{\frac{mv^2}{2}} = \frac{m + M}{m} \frac{v'^2}{v^2} = \frac{10.01}{0.01} \frac{0.998}{1000000} = \frac{1001}{1000} \frac{0.998}{1000} = 0.00998998$$

ANS: 99.9% of the initial energy is lost in the collision

Answer 5

For the block M the following is true:

$$Mg - T - n = 0$$

$$Mg - T = 0 \Rightarrow T = Mg$$

With the second equation describing moment when the M is about to move up.

For the swinging mass the following is true at the lowest point ( when the v is maximum and so the T is max)

$$T - mg = m \frac{v^2}{l}$$

$$\frac{(M - m)}{m} gl = v^2 \Rightarrow \frac{2.4 - 1}{1} (9.8)(0.6) \Rightarrow v^2 = 8.232 \left( \frac{m}{s} \right)^2$$

Using the conservation of energy for the swinging mass together with the above result yields:

$$\frac{mv^2}{2} = mgh \Rightarrow h = \frac{v^2}{2g} = \frac{8.232}{2(9.8)} = 0.42m = 42cm$$